electronic analog functions as an inseparable part of the automatic control system, making it possible to shape the required inputs.

The author expresses his gratitude to R. A. Velershtein and N. V. Prolygina, his co-workers in the Oscillations Laboratory, who rendered valuable assistance in the completion of the experiment.

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Reviewer's Comment

This paper describes an analog computer technique for systems analysis for determining a "worst worst-case." The reviewer is not aware of the application of this technique in this country, although alternate methods are employed to obtain equivalent design information. Therefore, the paper is a useful addition to the literature.

The author employs a set of "adjoint equations" to obtain intermediate results. It should be noted that his use of the term "adjoint" differs from the use of the same term in this country. As applied by Laning and Battin[1] and others,[2–4] the "adjoint method" is used for determining weighting functions for time-varying linear systems. The latter are closely related to the rms response of a system to random inputs.

One distinguishing difference of Savinov's development is his emphasis on worst-case performance. In this, he is close to some of the techniques used in analysis for optimum control systems (cf. Chang[5]). The example for the "hit" problem resembles a contactor or "bang-bang" servo system.

The extension to nonlinear systems is of particular interest. Conceivably these computer solutions might be implemented in fast-time to permit on-line optimization of a system or process.

—T. W. Connolly
Control Technology, Inc.

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Reliability Computation of Complex Automated Systems

A. M. Polovko

Computing the reliability of a system means determining its quantitative reliability parameters from the known parameters of the elements which make up the system.

A complete description of reliability can be given in terms of the probability of operation without failure $P(t)$, average time of reliable operation T_{av}, and the likelihood of failure $\lambda(t)$. The most widely used methods of computing the foregoing parameters are based on the assumption that the failures have a Poisson distribution and that an exponential reliability law applies.[1] In this case, to compute $P(t)$, T_{av}, and $\lambda(t)$ of the system, the likelihood of failure must be known for all elements of the basic system. Since the likelihood of failure of the elements depends essentially on their schedules and usage, one must have a family of curves which will determine the likelihood of failure as a function of the loading factors, the temperature of the ambient medium, the amplitude and frequency of vibrations, humidity, etc. Such relations are not available at present for most elements of automated systems. Further, the forementioned computational methods cannot be applied to the calculation of quantitative reliability parameters under various conditions of usage of the same system and make the formulation of reliability criteria difficult for separate parts and units of a complex system, and also lead to great computational errors. The coefficient method given here for computing the reliability of complex systems is free, to some extent, of the forementioned difficulties.

The method is based on the following assumptions: The failures are random and independent events; the failure of any element leads to failure of the entire system; the likeli-

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hood of failure $\lambda(t) = \text{const}$; the likelihoods of failure λ of all elements of an automated system vary, depending on the conditions of usage, in the same degree.

The first three assumptions mean that an exponential reliability law holds, and that the following relations are valid:[1]

$$P(t) = \exp\left(-t\sum_{i=1}^{n}\lambda_i\right) \qquad \lambda = \sum_{i=1}^{n}\lambda_i$$
$$T_{\text{av}} = \frac{1}{\sum_{i=1}^{n}\lambda_i} \tag{1}$$

where λ_i is the likelihood of failure of elements of type i, and n is the number of elements in the automated system.

The fourth assumption means that for different conditions of usage the relation

$$\lambda_i/\lambda_0 = k_i = \text{const} \tag{2}$$

holds, where λ_0 is the likelihood of failure of an element of the automated system whose quantitative reliability parameters are known with certainty. The element with likelihood of failure λ_0 will henceforth be called the principal element of the system, and k_i, the reliability coefficient of the ith element.

Such an assumption is valid with a high degree of accuracy for most elements of automated systems. For example, it is known[2] that the dependence of the likelihood of failure of the resistors λ_R and the capacitors λ_c on the loading factors k_1 can be approximated by expressions of the form

$$\lambda_R = a_R k_1[(b_R k_1)^2 + 1]^{n_R k_1}$$
$$\lambda_c = a_c k_1[(b_c k_1)^2 + 1]^{n_c k_1}$$

The coefficients b_r and b_c are less than unity for different types of resistors and capacitors. Therefore for $k_1 < 1$ the following expression holds:

$$k_i = \frac{\lambda_c}{\lambda_R} = \frac{a_c}{a_R} = \text{const}$$

Assuming that all elements of the same type have equal reliability and taking (2) into consideration, we will write expression (1) in the form

$$P(t) = \exp\left(-t\lambda_0\sum_{i=1}^{m}k_i n_i\right) \qquad \lambda = \lambda_0\sum_{i=1}^{m}k_i n_i \tag{3}$$
$$T_{\text{av}} = \frac{1}{\lambda_0\sum_{i=1}^{m}k_i n_i} = \frac{T_{\text{av},0}}{\sum_{i=1}^{m}k_i n_i}$$

where n_i is the number of elements of type i, m is the number of element types, and $T_{\text{av},0} = 1/\lambda_0$ is the average operating time without failure of the system as determined by the failures of the principal element.

It can be seen from expression (3) that to compute the quantitative reliability parameters one need not know the reliability of the elements of the automated system. It is sufficient to know only the reliability coefficients k_i, the number of elements in the system, and the likelihood of failure of the principal element.

The reliability coefficients k_i can be computed from the data pertaining to the likelihood of failure of the elements which are obtained from the usage of various automated systems.

Since the likelihood of failure of the elements depends on their performance, the qualifications of the servicing personnel, etc., the coefficients k_i which are thus obtained will lie within definite limits for the same elements. In this case it is useful to compute $P(t)$, $\lambda(t)$, and T_{av} for maximum and minimum values of k_i.

The relations $P = f(\lambda_0 t)$ for $k_{i,\min}$ and $k_{i,\max}$ are given in Fig. 1. It is evident from this figure that the probability

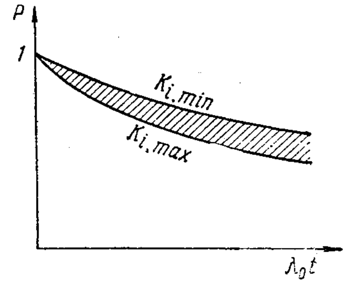

Fig. 1

of failure-free performance of the system will lie inside the shaded region. These curves make it possible to carry out a very simple recomputation of the quantitative reliability parameters under altered conditions of usage of the automated system.

In fact, with the condition $k_i = \text{const}$, only the scale of the curve $P = f(\lambda_0 t)$ changes under altered conditions of usage of the automated system. The magnitude of the change in scale can be computed if the relations between the likelihood of failure of the principal element and the conditions of usage are known.

Based on an analysis of domestic and foreign data[1] pertaining to the likelihood of failure of computers, shipboard systems, and aviation equipment, maximum and minimum values of the reliability coefficients were worked out for the elements given in Table 1.

Resistors were taken as the principal element. The values k_i which deviate considerably from $k_{i,\min}$ and $k_{i,\max}$ are excluded from the table. Such individual values k_i were considered unreliable.

In computing the reliability of a particular automated system, the reliability coefficient table must be made more precise. Greater precision is absolutely necessary in the case when special measures are taken in the system to increase reliability, such as an easing of the operating conditions of various elements.

The method makes it possible to compare the reliability of systems or of units of the same system with good accuracy merely on the basis of approximate data about the quantitative parameters of the elements.

In fact, for two systems or for units of the same system, we can, according to (3), write

$$P_1(t) = \exp\left(-t\lambda_0\sum_{i=1}^{m_1}k_i n_{i,1}\right)$$
$$\lambda_1 = \lambda_0\sum_{i=1}^{m_1}k_i n_{i,1} \qquad T_{\text{av},1} = \frac{T_{\text{av},0}}{\sum_{i=1}^{m_1}k_i n_{i,1}}$$
$$P_2(t) = \exp\left(-t\lambda_0\sum_{i=1}^{m_2}k_i n_{i,2}\right)$$
$$\lambda_2 = \lambda_0\sum_{i=1}^{m_2}k_i n_{i,2} \qquad T_{\text{av},2} = \frac{T_{\text{av},0}}{\sum_{i=1}^{m_2}k_i n_{i,2}} \tag{4}$$
$$\frac{T_{\text{av},1}}{T_{\text{av},2}} = \frac{\lambda_2}{\lambda_1} = \frac{\sum_{i=1}^{m_2}k_i n_{i,2}}{\sum_{i=1}^{m_1}k_{i,}n_{i,1}}$$
$$\frac{\ln P_1(t)}{\ln P_2(t)} = \frac{\lambda_1}{\lambda_2} = \frac{T_{\text{av},2}}{T_{\text{av},1}} = \frac{\sum_{i=1}^{m_1}k_i n_{i,1}}{\sum_{i=1}^{m_2}k_i n_{i,2}}$$

It is evident from these expressions that the reliability of systems can be compared if the composition of their elements and their reliability coefficients are known. In this connec-

Table 1

Name of element	$k_{i,\,\min}$	$k_{i,\,\max}$	Name of element	$k_{i,\,\min}$	$k_{i,\,\max}$
Electropneumatic devices	18.3	26.6	Selenium and cuprous oxide rectifiers	16.7	20
Oscillator tubes	70	77	Electric motors	17	22
Capacitors	0.33	0.61	Converters	70	100
Tantalum capacitors	10.7	15.3	Relays	3.3	5.5
Resistors	1	1	Gyroscopes	97.5	100
Potentiometers	7.2	12	Delay lines	62.5	93.4
Semiconductor diodes	11.7	15.4	Plug connectors	10.7	15.3

tion, it is not necessary to know the quantitative reliability parameters of the elements, including also the principal element. Since the ratios $\sum_{i=1}^{m_1} k_i n_{i,1} / \sum_{i=1}^{m_2} k_i n_{i,2}$ vary less significantly than do the coefficients $k_{i,\,\min}$ and $k_{i,\,\max}$, the reliability comparison of a system or of its units is carried out with good accuracy.

Using the coefficient method of computing reliability in the initial stages of designing a complex system, we can simplify computations considerably by requiring that the quantitative reliability parameters of the separate units have the form $\sum_{i=1}^{m} n_i k_i$.

In fact, expression (3) for the probability of failure-free performance can be written in the form

$$P(t) = \exp\left\{-t\lambda_0\left[\sum_{i=1}^{m_1} k_i n_{i,1} + \sum_{i=1}^{m_2} k_i n_{i,2} + \ldots + \sum_{i=1}^{m_j} k_i n_{i,j}\right]\right\} \quad (5)$$

where $m_1, m_2, \ldots, m_j$, is the number of element types in blocks 1, 2, ... j, and $n_{i,1}, n_{i,2}, \ldots, n_{i,j}$, is the number of elements of type i in blocks 1,2,... j.

Knowing the requirement for the probability of failure-free work $P(t)$ in the time interval t, and the likelihood of failure of the principal element, we obtain from (5)

$$\sum_{i=1}^{m_1} k_i n_{i,1} + \sum_{i=1}^{m_2} k_i n_{i,2} + \ldots + \sum_{i=1}^{m_j} k_i n_{i,j} = -\frac{\ln P(t)}{t\lambda_0}$$

If no steps are taken to insure reliability, then

$$\sum_{i=1}^{m_1} k_i n_{i,1} + \sum_{i=1}^{m_2} k_i n_{i,2} + \ldots + \sum_{i=1}^{m_j} k_i n_{i,j} > -\frac{\ln P(t)}{t\lambda_0}$$

or

$$a_1 \sum_{i=1}^{m_1} k_i n_{i,1} + a_2 \sum_{i=1}^{m_2} k_i n_{i,2} + \ldots + a_j \sum_{i=1}^{m_j} k_i n_{i,j} = -\frac{\ln P(t)}{t\lambda_0}$$

where $a_1, a_2, \ldots, a_j$, are coefficients, most of which are less than unity.

The values of these coefficients can be determined, if we require that the system satisfy the following qualitative criterion.

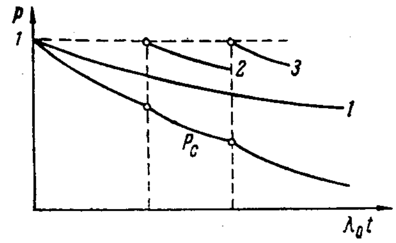

Fig. 2

The system is optimal in the reliability sense if it satisfies the requirements for the quantitative reliability parameters and is constructed such that units of equal complexity have equal reliability.

In accordance with this criterion, coefficients $a_1, a_2, \ldots, a_j$, must be distributed among the blocks in proportion to their complexity. This makes it possible to express the technical requirements for all blocks of the system not in the form of the probability of failure-free performance or some other quantitative reliability parameter, but in the form of a sum $\sum_{i=1}^{m_k} k_i n_{i,k}$.

Having a table of reliability coefficient values for the elements, the designer can select the element types and their number in a given arrangement. If it turns out that the arrangement which satisfies a given value of a sum of the form $\sum_{i=1}^{mk} k_i n_{i,k}$ cannot be met, then this will mean that special steps must be taken to increase the reliability of the block. Such steps can be a reduction in the reliability coefficients by easing the operating conditions of the elements, the use of standby equipment, etc.

In computing reliability by the methods cited it is expedient to calculate the probability of failure-free performance by blocks and construct the curves $P(\lambda_0 t)$ for all blocks of the system on a single graph. This makes possible a graphic comparison of blocks according to reliability, the detection of weak spots in the system, and the outlining of ways to improve reliability.

A characteristic feature of complex automated systems is that not all of its blocks work continuously from the moment the system is started to the moment it is disconnected.

During various time intervals one or another part of the system can be working. Therefore curves $P = f(\lambda_0 t)$ which were constructed for blocks can be displaced relative to time.

To compute the probability of failure-free performance of the system in this case, we must use the multiplication rule for conditional probabilities. The probability of failure-free performance of system $P_c = f(\lambda_0 t)$, which has thus been computed, will no longer vary exponentially (Fig. 2), although an exponential reliability law is valid for the separate blocks of the system. The computations become more complicated if the curves $P = f(\lambda_0 t)$ of individual blocks intersect. Hence, in computing the quantitative reliability parameters we must take into account the nonsimultaneity in the work of the units; otherwise the computation will yield a lower value for the probability of failure-free performance.

The selection of the number of elements of the system which must be taken into account in computing the reliability is also important. It is often the case in complex automated

Table 2

Element type 1...m	Block 1					Block 2				
	n_i	$k_{i,\max}$	$n_i k_{i,\max}$	$k_{i,\min}$	$n_i k_{i,\min}$	n_i	$k_{i,\max}$	$n_i k_{i,\max}$	$k_{i,\min}$	$n_i k_{i,\min}$
$\sum_{i=1}^{m} n_i k_i$			$\sum_{i=1}^{m} n_i k_{i,\max}$		$\sum_{i=1}^{m} n_i k_{i,\min}$			$\sum_{i=1}^{m} n_i k_{i,\max}$		$\sum_{i=1}^{m} n_i k_{i,\min}$

systems that they contain elements whose failure leads merely to deterioration of certain parameters of the system (precision, the quality of a transient response, etc.). The failure of other elements impairs the efficiency of the system, i.e., in the reliability sense the elements are not equivalent. In computing the reliability, only those elements must be taken into account whose nonfunctioning leads to failure.

Thus, before computing reliability, we must define rigorously what is meant by the failure of the system.

In using the coefficient method for computing the reliability of complex automated systems, it is useful to adopt the following order of computation.

1) Formulate the concept of failure for the given system.

2) Construct a plan for computing the reliability (a set of elements representable in graph form, and indicate how these are connected in the reliability sense). In the computational plan, show the time interval for the work of every element of the computation. It is expedient to divide the elements into groups, according to their working times, and to form these groups into the elements of the computation.

3) Select the principal element of the system, i.e., an element whose likelihood of failure is known with certainty. Most often such elements are resistors, capacitors, inductors, etc.

Let us assume that, for the computation, a resistor was taken as the principal element. Then it may turn out that the automated system consists of resistors of various types (MLT, ULM, VS, etc.), whereas resistors of the same type are distinguished by nominal values of the principal parameter. In this case it is expedient to find a weighted mean value of the likelihood of failure of all resistors of the system and to take this value as the principal value (λ_0).

Clearly the λ of the resistors in the blocks will be different from λ_0, and the mean weighted reliability coefficients of the resistors in the various blocks will be different from unity.

4) Construct a computational table (Table 2).

In Table 2, n_i is the number of elements of type i in the block which lead to failure as defined in point 1; $k_{i,\max}$ and $k_{i,\min}$ are the maximum and minimum values of the reliability coefficients of the elements, taken from Table 1.

5) From the data of Table 2, construct for each block the relations $P = f(\lambda_0 t)$ for the maximum and minimum values of the reliability coefficients of the elements.

6) Construct a table of relative values of the mean time of failure-free performance of the blocks. The ratios are computed from formulas (4) and the data of Table 2.

Table 3

Name of block	block 1	block 2	block 3	...	block n
$\frac{T_{av,i}}{T_{av,1}}$	1	0.8	0.6	...	0.95

Table 3 provides a graphic comparison of the reliability of the blocks and enables us to pinpoint the least reliable blocks of the system.

7) From the data of point 5, construct for $k_{i,\max}$ and $k_{i,\min}$ curves for the probability of failure-free performance of the automatic system as a function of $\lambda_0 t$.

8) From the known time t of continuous work of the system and the likelihood of failure of the principal element λ_0, compute the probability of failure-free performance of the automated system.

9) From the known probability of failure-free performance compute, using formulas (1), the average failure-free performance time and the likelihood of failure of the system.

If the curve $P = f(\lambda_0 t)$ has discontinuity points, then the parameters are computed using formulas (1):

$$T_{av} = \int_0^\infty P(t)dt$$

$$a(t) = -P'(t) \qquad \lambda(t) = a(t)P(t)$$

The data which were obtained as a result of the computation are compared with those required, and a conclusion is then reached about the work of the system in the sense of its reliability.

—Submitted May 25, 1960

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Reviewer's Comment

A method of calculating the reliability of a system is presented which is not startling and does not portray any powerful mathematical tool. The author describes the standard reliability formulas which have appeared countless times in the literature and which are the common foundations for all reliability work. However, he does show an interesting technique that has been overlooked by most reliability engineers, who are striving constantly for hairline accuracy in their calculations. This scheme is presented very clearly and is well summarized. It would make a valuable addition to any tutorial symposium or reliability training session.

Instead of using absolute values of failure rates for parts, which is the usual American practice, a table of relative minimum and maximum constants is used. This constant value technique has been developed because of the unavailability of mean failure rate data. These constants are directly related to some principal part whose likelihood of failure is known, i.e., a resistor with a k of unity.

Since the constants are both minimum and maximum, the conclusion gives a relative spread or estimated reliability band.

When this technique is used for a single system, it does not have much value because its true failure rate λ is difficult to reclaim. When used as a method of comparing different systems or designs, it is handy and worth while. Most American companies have developed curves and tables that present the failure rates of parts as some function of stress. However, in many cases, these failure rates are not true estimates of the mean and are not accurate. When only limited data are available, it might be better to show the system reliability as a spread or band as the author proposes.

—M. Raphelson
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